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Numerical Simulation of Magnetohydrodynamic (MHD) Fluid Flow Over A Stretching Surface

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Abstract
An incompressible electrically conducting fluid's constant two-dimensional magnetohydrodynamic (MHD) boundary layer flow across a continuously extended surface is numerically examined in great depth in this study. These flows are often used in applications such as molding polymers, rolling metal, making glass fiber, cooling stretched sheets, and MHD-based flow control devices. When a transverse magnetic field is applied, Lorentz forces are exerted, and momentum transfer in the boundary layer is drastically altered. In this research, the momentum and mass conservation nonlinear partial differential equations are expressed in terms of boundary layer approximations, and then, by use of similarity transformations, they are reduced to a set of nonlinear ordinary differential equations. A fourth-order Runge Kutta method is used numerically to solve these modified equations by a shooting strategy. The effect of important physical factors, such stretching and magnetic ones, on the velocity distribution and thickness of the boundary layer is studied in detail. Important insights into MHD flow control mechanisms are provided by the quantitative results, which demonstrate that the extension surface increases the near-wall velocity and the working magnetic field inhibits fluid flow.

Keywords: Magnetohydrodynamics, Stretching surface, Boundary layer flow, Similarity transformation, Numerical simulation

Introduction

A subject of substantial scientific relevance, the flow of a fluid across a stretched surface has inherent importance within fluid mechanics and strong connections to several practical engineering and commercial applications. The fluid is drawn in closer by the constantly moving surface, which causes a dynamic interaction that changes the boundary layer's thickness, shear stress distribution, and velocity field, in contrast to flow on a static or rigid surface. The stretching-induced motion is more nonlinear and produces significantly different flow characteristics than the typical boundary layer flows.

The expansion of the surface causes a change in the flow characteristics, which in turn creates complex boundary layers. These layers have a major influence on the surface's mass, heat, and momentum transport capabilities. In industrial operations including wire drawing, coating, metal forming, and polymer extrusion, these play a crucial role in regulating fluid flow for efficient operation and high-quality products. In order to construct trustworthy theoretical and numerical models, precisely forecast fluid behavior under real-world industrial operating conditions, and optimize the efficiency of different processes, it is crucial to have a solid grasp of the mechanics of boundary layer flow over stretching surfaces.

Most manufacturing and materials processing operations rely on the stretching surface's motion to control the flow of a surrounding fluid. This includes processes such as continuous casting, drawing and coating copper and metallic wires, rolling and shaping metal plates, producing glass fiber and sheets, and extruding plastic and polymer sheets. The process defines the boundary layer's thickness and velocity field as a result of the fluid-surface interaction, which influences the near-surface mass, heat, and momentum transfer. In order to regulate the cooling rates, coating placement, and solidification throughout the production process, these transportation techniques are crucial.

The surface polish, structural integrity, and dimensional accuracy of the manufactured goods are significantly affected by the shear stress and velocity gradients that are generated around the stretched body. Even a little change in flow behavior could lead to undesirable effects including uneven thickness, surface flaws, residual tensions, or uneven material properties. In addition to lowering the product's aesthetic value, these defects may compromise its technical attributes and reliability over time.

Research on boundary layer flow across stretched surfaces may, therefore, substantially contribute in understanding the properties of this flow and enhancing process control strategies in many industrial applications. Having a complete grasp of the flow behavior close to the stretching surface would allow engineers to control important parameters like stretching speed, surface and fluid properties, and working conditions, resulting in the desired heat transfer, surface finish, and constant material. In highly precise manufacturing processes, where even little changes in flow may cause major defects or inefficiency, this kind of data is crucial.

Engineers may improve energy efficiency, decrease manufacturing costs, and reduce material waste by precisely modeling and evaluating the boundary layer flows behind the stretched surfaces. This allows them to optimize fluid-surface interactions and avoid superfluous resistance. Maintaining consistent high-quality items and better mechanical service is possible with the use of trustworthy flow models that can foretell when defects will develop and how to prevent them. So, to improve current manufacturing technologies, create new engineering systems that are both effective and sustainable, and advance the theory of industrial fluid dynamics, more study on stretched surface boundary layer flows is required.

Role of Magnetohydrodynamics

When an electrically conducting fluid is exposed to an external magnetic field, the magnetohydrodynamic (MHD) effects become substantial because of the intimate connection between electromagnetic forces and fluid velocity. As a conductor fluid flows through a magnetic field, it generates electric currents in accordance with the principles of electromagnetic induction. Lorentz forces act against the fluid's velocity when the applied magnetic field interacts with these produced currents. This force of resistance changes the structure of the boundary layer, slows the fluid down, and affects the internal momentum transit.

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Adding a magnetic field provides an extra way to control and manage the flow behavior, even when there isn't direct physical contact. Regulating velocity profiles, reducing boundary layer thickness, and suppressing velocity shifts may be achieved by suitably altering the magnetic field's strength. Scientists have shown that this magnetic impact helps to reduce flow turbulence and instabilities, making for a more stable and peaceful flow environment. Since this kind of control is difficult to do with the help of traditional mechanical techniques, MHD is a fantastic complement to the modern approach to flow management.

For engineering systems operating at high speeds and temperatures, MHD flow control is an excellent alternative to more traditional methods of controlling flow. In metallurgical processes, a magnetic field is used to regulate the movement of molten metal, provide better mixing, and elevate the quality of the final product. The use of MHD effects in nuclear reactors aids in the regulation of coolant flow and the preservation of thermal stability under extreme conditions. Aerospace and plasma-based systems use magnetic fields for similar purposes, stabilizing boundary layers, lowering thermal loads, and controlling flow behavior in highly heated and electromagnetically-interacting environments. Therefore, the optimization and design of contemporary energy, materials processing, and high-performance engineering systems rely heavily on MHD effects on boundary layer flows.

Mathematical Formulation

In order to study how the boundary layer is affected by the surface stretching effect and magnetic influence, a mathematical model that adequately describes the physical situation has to be created. Based on a set of physically valid and generally acknowledged assumptions, the mathematical models presented here attempt to simply the fundamental mechanisms driving fluid movement within the context of electromagnetic forces, which is an inherently complex interaction. An approach like this might be useful for capturing the basic features of magnetohydrodynamic (MHD) boundary layer flows. Modeling the flow geometry, fluid characteristics, and external forces allows for a consistent and systematic approach to deriving the governing equations. After the equations are converted, the similarity technique allows for numerical solutions with excellent accuracy.

This leads us to believe that the x-axis is perpendicular to any two-dimensional, laminar, stable, incompressible, electrically conducting Newtonian fluid flow. The flow is produced when the surface extends linearly in its plane, resulting in a relationship between the surface's velocity and the distance to the origin. The majority of stretching and extrusion processes in industry are quite comparable to this state. This extensional action continuously draws the fluid down the side, forming a boundary layer. The boundary layer's structure is greatly affected by the extension rate. When a constant force B 0 is applied uniformly and perpendicular to the surface in the y-direction, the flow undergoes a magnetohydrodynamic effect. As the conducting fluid passes through the magnetic field, it generates electric currents.

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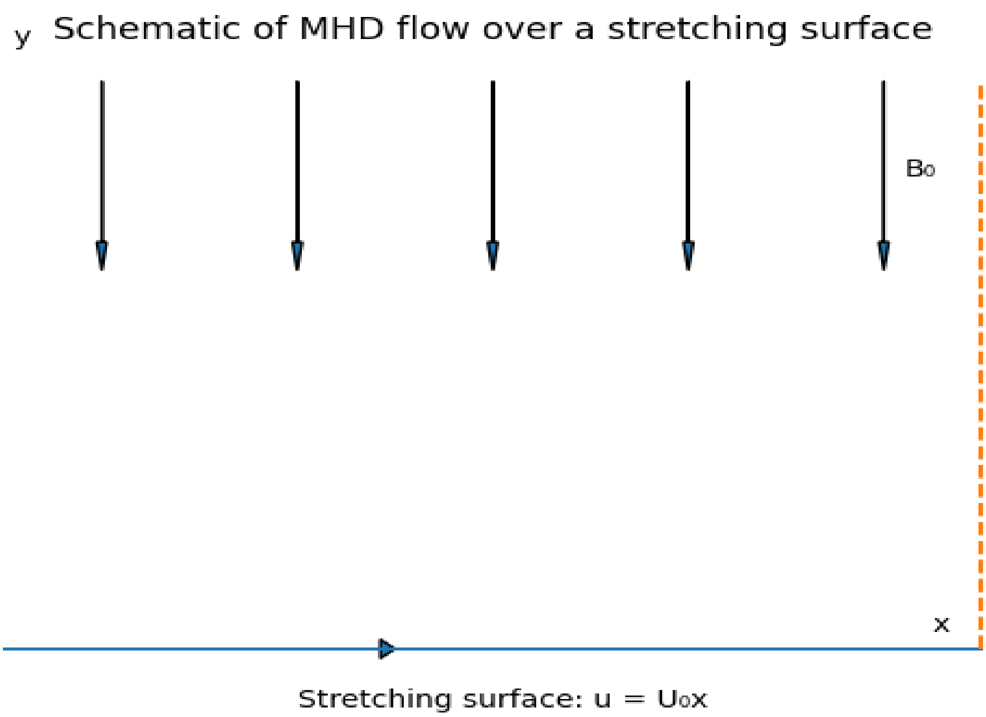

Figure 1: schematic of MHD flow over a stretching surface

By combining with the applied magnetic field, these currents produce Lorentz forces, which alter the momentum transferred in the boundary layer and counteract the fluid's movement. These currents are believed to generate a small induced magnetic field due to the low magnetic Reynolds number, which suggests that magnetic diffusion is greater than magnetic induction. If the applied magnetic field is considered to be externally applied and totally independent of the flow, then the analysis may be simplified and the necessary magnetic damping effects on the velocity field and the development of the boundary layer can be obtained.

Physical Model Description

For simplicity, let's pretend that an x-axis and an elastic, smooth surface are encircled by a two-dimensional boundary layer through which a Newtonian fluid, which is both incompressible and electrically conductive, flows continuously and laminarly. The commencement of flow is caused by the surface's linear expansion along its own plane, where its velocity is directly proportional to the distance from the origin. The fluid's stretching rate primarily determines the thickness and velocity structure of the boundary layer that is generated as a result of the continual drag of the fluid via the stretch mechanism. These flow patterns are very useful in the real world because they show how flows behave near surfaces, which is important for industrial operations that involve continually stretching sheets, films, or filaments.

The impact of surface stretch on the locally balanced momentum causes noticeable differences in the velocity gradients seen in flows over smooth versus motionless surfaces. Due to its potential for self-similar behavior under certain conditions, the boundary layer is an excellent choice for analytical and numerical studies. The surface's elasticity, which makes it continually deform and move, also affects the flow because it increases the surface-fluid contact area.

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Consideration of the electromagnetic effect requires positioning a surface that is to be stretched in a homogeneous magnetic field of constant strength B 0, perpendicular to the Y-direction. Magnetohydrodynamic (MHD) effects occur when a magnetic field is present; these effects induce electric currents to flow through a fluid that conducts electricity. The produced currents and the applied magnetic field provide justification for the Lorentz forces, which operate in the inverse direction of the fluid's movement. The fluid's velocity is reduced and the boundary layer's thickness is altered as a result of this magnetic resistance changing the momentum transfer in the boundary layer.

In order to regulate the flow properties, the magnetic field effect provides a practical, non-contact technique. The strength of the magnetic field may be controlled to inhibit boundary layer flow instabilities and control the velocity profiles. Because of the small magnetic Reynolds number, the induced magnetic field from moving fluid is thought to be negligible and driven mostly by magnetic diffusion and not induction. The applied magnetic field is still under control, therefore the flow won't affect it. While keeping the important physical process of magnetic damping, this substantially simplifies the mathematical model for studying MHD boundary layer flows on stretched surfaces. The model becomes analytically solved and gains scientific significance as a result.

Governing Equations

Under boundary layer approximations, the governing equations for the flow are:

(1) Continuity equation

$$\partial u/\partial x + \partial v/\partial y = 0 \qquad (1)$$

(2) Momentum equation in the x-direction:

$$u(\partial u/\partial x) + v(\partial u/\partial y) = \nu(\partial^2 u/\partial y^2) - (\sigma B_0{}^2/\rho)u \qquad (2)$$

where u and v are velocity components in the x and y directions respectively, ν is the kinematic viscosity, σ is electrical conductivity, ρ is fluid density, and the last term represents the Lorentz force.

Boundary Conditions

The boundary conditions demonstrate the physical constraints of the flow at both the stretched surface and the free stream, which is located distant from the surface. The surface's boundary conditions account for the no-slip and impermeability requirements, which guarantee that the fluid velocity is equal to the surface's stretching velocity and that the surface does not allow normal flow. Under these circumstances, the fluid layer around the elastic surface interacts with its motion.

By adjusting the boundary conditions of the far-field area, we may make the flow velocity approach that of the ambient fluid as the surface distance rises. Once the effects of the stretched surface and magnetic forces reduce enough, the physical expectation is that the fluid will be able to return to its undisturbed state at the boundary. The mathematical problem is given a well-posed physical issue and the mathematical framework is necessary to provide numerically stable solutions that are well-posed and physically relevant by virtue of these boundary conditions taken together.

$$u = U_0 x, v = 0 \; at \; y = 0 \quad (3)$$

$u \to 0$ as $y \to \infty$

Similarity Transformation

Finding a practical and efficient solution to the governing flow equations requires reducing the mathematical complexity of the issue without compromising its important physical features. The nonlinear partial equations that characterize boundary layer equations are notoriously hard to resolve. By using similarity transformation methods, we are able to circumvent this obstacle and reframe the flow problem in a way that is simpler and more physically meaningful. This method works well with flows over stretched surfaces, where, with the correct assumptions, self-similar behavior may be taken advantage of.

By using similarity transformations, the governing partial differential equations are simplified and become more calculable. If you have partial differential equations, you may use similarity analysis to turn them into ordinary differential equations by combining many independent variables into one similarity variable. This transformation not only adds dimension to the problem, but it also provides an opportunity to employ robust numerical approaches to investigate the impact of key physical parameters on the flow behavior in a systematic and effective manner.

Purpose of Similarity Analysis

A powerful analytical tool to simplify the governing partial differential equations and decrease computing complexity is suggested in the form of similarity transformations. Because they are often nonlinear and include several independent variables, boundary layer equations may be challenging to solve analytically or numerically directly. A possible solution to this problem is to use similarity analysis to find the right similarity variable; this will allow us to combine the spatial coordinates into one independent variable and see how self-similarity behaves in the flow field. Because of this, non-dimensional functions of the similarity variable may be used to describe velocity components and other dependent variables.

Consequently, the mathematical complexity and size of the problem are drastically decreased from a system of partial differential equations to a system of ordinary differential equations. Considering the importance of the most important dimensionless parameters controlling the flow, the transformation streamlines the solution procedure and enhances the use of the physical interpretation. Effective numerical approaches based on reduced ordinary differential equations also allow for the precise and systematic study of the effects of many physical elements on boundary layer behavior.

Stream Function Formulation

A stream function ψ (x, y) is introduced such that:

$$u = \partial\psi/\partial y \quad (4)$$

$$v = -\partial\psi/\partial x \quad (5)$$

This formulation automatically satisfies the continuity equation (1).

Similarity Variables

The similarity variable η and stream function ψ are defined as:

$$\eta = \sqrt{(U_0/\nu)}\, y \quad (6)$$

$$\psi = \sqrt{(\nu U_0)}\, x\, f(\eta) \qquad (7)$$

Substituting equations into the momentum equation yields a nonlinear ordinary differential equation.

Similarity Equation

The resulting similarity equation is:

$$f''' + f\,f'' - (f')^2 - M\,f' = 0 \qquad (8)$$

where prime denotes differentiation with respect to η and $M = \sigma B_0^2/(\rho U_0)$ is the magnetic parameter.

Transformed Boundary Conditions

The boundary conditions become:

$$f(0) = 0, f'(0) = 1 \qquad (9)$$

$$f'(\infty) \to 0 \qquad (10)$$

Numerical Methodology

The nonlinear ordinary differential equation, which arises from applying similarity transformations and the related boundaries on the surface and in the far field, creates a classical boundary value problem. Linked nonlinear elements are at the heart of this question's dilemma due to the convective acceleration and magnetic effects, which complicate the governing equation. These nonlinearities make it impossible to apply standard mathematical methods to generate a closed form analytical solution.

Consequently, a numerical method is necessary for obtaining valid solutions to the altered equations. It is feasible to compute the result with controlled accuracy and solve complicated boundary conditions and strong nonlinear behavior numerically. By using suitable numerical methods, one may thoroughly examine the flow properties, including velocity profiles, and the variables that influence them.

Shooting Technique

The shooting method is employed to transform the given boundary value problem into an equivalent initial value problem, which is more convenient and efficient for numerical implementation. In boundary layer analyses, the governing ordinary differential equations are typically accompanied by boundary conditions specified at two different points—at the surface and at infinity—making direct numerical solution challenging. The shooting technique overcomes this difficulty by converting the problem into an initial value formulation, where all required conditions are specified at the starting point.

In this approach, the unknown boundary conditions at the surface, such as the second derivative of the dimensionless stream function $f''(0)$, are treated as adjustable or "shooting" parameters. An initial estimate for these unknown quantities is assumed, and the resulting system of coupled ordinary differential equations is integrated numerically from the surface toward the far-field using a suitable numerical integration scheme, such as the Runge–Kutta method. This forward integration generates a trial solution over the computational domain.

The computed solution is then evaluated to determine whether it satisfies the prescribed boundary conditions at infinity within an acceptable error tolerance. If the far-field conditions are not met, the initial guess for $f''(0)$is systematically refined using an iterative procedure,

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such as a trial-and-error approach or a root-finding algorithm. This iterative process is repeated until convergence is achieved and the numerical solution satisfies all boundary conditions with the desired accuracy. Owing to its flexibility and reliability, the shooting method is particularly effective for boundary layer problems, as it offers stable and accurate solutions while allowing precise control over convergence behavior and numerical accuracy.

Runge–Kutta Integration

The fourth-order Runge-Kutta method, famous for its stability, robustness, and high accuracy, is numerically used to handle an initial value problem that arises from solving a system of ordinary differential equations. It strikes an affordable compromise between the need for precise calculations and the associated expenditures. Thanks to its systematic integration approach, the solution may be gradually transferred from the surface to the far field with excellent control over numerical inaccuracies.

To guarantee convergence and correctness of the numerical solution, the boundary conditions are subject to tight tolerance limits, and the step size is meticulously set and improved. Iteratively carrying out the integration operation until the calculated results from consecutive integrations show hardly any variance, signifying convergence. The consistency and dependability of the computed flow characteristics are a result of the meticulous refining process that ensures the numerical solution meets the required accuracy of the far-field boundary conditions.

Results and Discussion

For a range of magnetic parameter M values, numerical solutions are produced. To comprehend the function of electromagnetic forces, the impact of M on velocity profiles and boundary layer thickness is examined.

Effect of Magnetic Parameter

The fluid velocity across the boundary layer is greatly decreased by an increase in the magnetic parameter. The Lorentz force, which functions as a resistive force opposing the velocity of the electrically conducting fluid, is responsible for this behavior. Consequently, the suppression of momentum diffusion results in smaller boundary layers.

Physical Interpretation

The magnetic field effectively converts kinetic energy of the flow into thermal energy through Joule dissipation, thereby damping the velocity field. This property can be exploited in industrial processes where controlled flow deceleration is required.

Conclusion

This research is a numerical study of the steady magnetohydrodynamic (MHD) flow boundary in the steady flow past a stretching boundary. The process involved deducing the nonlinear partial equations that govern the flow, which were then reduced to a system of coupled ordinary linear equations using suitable similarity transformations. The Runge-Kutta numerical scheme and a robust shooting methodology were used to solve the system. Results show that the fluid's velocity profiles are lowered and the boundary layer is thinned down by the transverse magnetic field's generation of the Lorentz force, which in turn strongly retards the fluid's motion. Surface stretching, on the other hand, improves near-wall velocity motion via enhanced momentum transfer in the fluid. Thus, the boundary layer's structure and behavior are dictated by the fine-tuning of the flow behavior as a whole, which is controlled by the interplay between the acceleration due to stretch and the magnetic dampening effects. Moreover, the numerical and mathematical framework can be easily extended to include non-Newtonian fluid modeling, effects of porous media, thermal radiation, unsteady stretching surfaces, and heat and mass transfer, greatly increasing its applicability to a wide range of real-world engineering and industrial processes. There will be a plethora of future study possibilities made possible by this.

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