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A Novel Green Sustainable Economic Order Quantity (G-SEOQ) Model for Deteriorating Items with Preservation-Technology Investment and Carbon-Emission Cost in Green Supply Chain Management Systems

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Abstract: Amid increasing carbon regulations and a push for sustainable supply chains, this paper presents a novel Green Sustainable Economic Order Quantity (G-SEOQ) model for deteriorating items that incorporates both preservation technology investment and carbon-emission costs. The model unifies classical inventory costs, controllable exponential deterioration, convex green investment costs, and carbon-tax-based emissions from ordering, warehousing, procurement, and disposal. Analytical and numerical analysis shows the model is jointly convex in cycle time and green-investment level, ensuring a unique optimal solution. Comparative results indicate the proposed approach reduces total cost and carbon emissions relative to traditional and partially-green inventory models. Sensitivity analysis confirms that the optimal policy is robust to changes in environmental parameters, providing practical insights for green supply chain management.

Keywords: Deteriorating items; Preservation technology; Carbon emission; Carbon tax; convex optimisation.

1. Introduction

Inventory control is increasingly moving beyond traditional cost minimization to embrace environmental responsibilities, which is especially important for products subject to continuous deterioration. While classical EOQ models have been augmented to account for perishability, there is now added emphasis on incorporating carbon-tax policies and green technology investments, reflecting mounting environmental regulations and heightened consumer demand for sustainable practices. Despite these advances, integrated models that simultaneously optimize replenishment cycles, preservation-technology investment, and explicitly calculate the total cost of carbon emissions from all inventory activities remain rare. This paper aims to bridge this gap by proposing and evaluating a unified Green Sustainable Economic Order Quantity (G-SEOQ) model, offering comprehensive guidance for sustainable inventory decisions in modern green supply chain management.

2. Literature Review

Chen et al. (2013) proposed a carbon-constrained EOQ model to incorporate emission limits directly into inventory decisions, demonstrating that optimal order sizing changes significantly under environmental regulations and helps organizations plan within emissions targets. Ghosh

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et al. (2021) Developed supply chain coordination models for green products considering various carbon emission reduction technologies, emphasizing the importance of technology choice and joint decision-making to minimize both costs and emissions across the chain. Govindan et al. (2015) provided a comprehensive review of reverse logistics and closed-loop supply chains, highlighting strategies for integrating sustainable practices that involve waste recovery, remanufacturing, and reduced environmental footprints. He et al. (2015) Examined production lot-sizing decisions under carbon taxation and cap-and-trade regulations, showing how different policy mechanisms alter production schedules and total supply chain emissions. Hsu et al. (2010) Analyzed investments in preservation technologies for inventories prone to deterioration, illustrating how such investments prolong product shelf-life, reduce losses, and improve profit in perishable supply contexts. Hua et al. (2011) Studied carbon footprints within inventory management and showed that integrating carbon-related costs and constraints leads to different optimal policies compared to models focused solely on traditional financial costs. Hussain et al. (2025) Explored AI-based emission reduction technologies in sustainable supply chain management, demonstrating that artificial intelligence can support predictive emissions tracking and inform greener operational strategies. Jaouhar et al. (2024) reviewed the role of ICT in decarbonizing agricultural supply chains and found that digital technologies greatly enhance tracking, transparency, and emissions optimization in agri-food logistics. Mishra et al. (2020) developed a model for sustainable production-inventory management with a controllable carbon emissions rate under shortages, illustrating how emissions can be jointly optimized alongside profitability despite supply interruptions. Srivastava (2007) Delivered a state-of-the-art literature review on green supply-chain management, summarizing core principles and identifying the environmental, economic, and operational challenges and opportunities for integrating sustainability in modern supply networks. Tiwari et al. (2018) Modeled sustainable inventory management practices that include deterioration, imperfect quality, and carbon emissions, recommending integrated policies to manage product loss and emissions alongside classical financial objectives. Yassine et al. (2024) Provided a systematic review of ICT-enabled decarbonization strategies for agricultural supply chains, confirming the positive impact of information and communication technologies in reducing greenhouse gas emissions and improving sustainability reporting.

3. Problem Description

Consider a retailer or distribution center within a two-echelon green supply chain that acquires a single item from its supplier at a constant demand rate D, immediately selling or consuming it at the same rate. The held inventory is subject to continuous deterioration over time. To mitigate this, the retailer can invest in preservation (green) technology such as refrigeration, controlled environments, advanced packaging, or energy-efficient warehousing at a continuous rate ξ, thereby lowering the effective deterioration rate. Each replenishment cycle results in carbon emissions from four main sources: (i) transportation and ordering, (ii) energy usage for storage, (iii) production or procurement of incoming stock, and (iv) disposal of unsold, deteriorated items. A carbon tax is applied to the total emissions generated per unit time. The

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central decision is to jointly optimize the replenishment cycle time T and the level of green investment ξ in order to minimize the total relevant cost per unit time, factoring in both conventional costs and carbon emission charges.

4. Proposed Mathematical Model

4.1 Inventory dynamics with controlled deterioration

Let I(t) denote the on-hand inventory level at time $t \in [0, T]$. Under constant demand D and effective (controlled) deterioration rate θ(ξ), the inventory level depletes according to the classical Ghare–Schrader-type differential equation

$$dI(t)/dt + \theta(\xi)\cdot I(t) = -D, \quad 0 \le t \le T, \quad I(T) = 0 \tag{1}$$

where the controlled deterioration rate is modelled as

$$\theta(\xi) = \theta 0 \cdot e^{(-\delta\xi)}, \quad \xi \ge 0, \ \delta > 0 \tag{2}$$

Equation (2) captures the essential behaviour of preservation technology: at $\xi = 0$ the deterioration rate equals its natural (uncontrolled) value θ0, and as ξ increases, θ(ξ) decreases toward zero at a diminishing rate, reflecting the fact that additional green investment yields progressively smaller reductions in deterioration. Solving the linear first-order differential equation (1) subject to the boundary condition I(T) = 0 gives

$$I(t) = (D/\theta(\xi)) \cdot [\, e^{(\theta(\xi)(T-t))} - 1 \,], \quad 0 \le t \le T \tag{3}$$

The order quantity per cycle equals the initial inventory level, Q = I(0), i.e.

$$Q(T, \xi) = (D/\theta(\xi)) \cdot [\, e^{(\theta(\xi)T)} - 1 \,] \tag{4}$$

$$H(T, \xi) = \int_0^T I(t)\, dt = (D/\theta(\xi)^2)\cdot[e^{(\theta(\xi)T)} - 1] - D\cdot T/\theta(\xi) \tag{5}$$

The number of units lost to deterioration per cycle is simply $Q(T, \xi) - D\cdot T$, i.e. the gap between what is ordered and what is actually demanded over the cycle.

4.2 Total cost per unit time

The total relevant cost incurred over one replenishment cycle consists of six components: (i) a fixed ordering cost A, (ii) a purchasing cost c·Q, (iii) a holding cost h0·H(T, ξ), (iv) a deterioration (write-off/disposal) cost cd·(Q − D·T), (v) a strictly convex green/preservation-technology investment cost ρg·ξ², charged per unit time, and (vi) a carbon-emission cost, described in Section 4.3. Dividing the cycle cost by the cycle length T gives the total-cost-per-unit-time function

$$TC(T, \xi) = [A + c\cdot Q(T,\xi) + h0\cdot H(T,\xi) + cd\cdot(Q(T,\xi) - D\cdot T) + Cec(T,\xi)] / T + \rho g\cdot \xi^2 \tag{6}$$

where Cec(T, ξ) is the total carbon-emission cost per cycle, defined next.

4.3 Carbon-emission function

The net physical carbon emission generated per cycle is the sum of the four emission streams introduced in Section 3.2:

$$Ec(T, \xi) = eo + eh\cdot H(T,\xi) + ep\cdot Q(T,\xi) + ed\cdot(Q(T,\xi) - D\cdot T) \tag{7}$$

and the corresponding carbon-emission cost per cycle, charged at the fixed carbon-tax rate Ct, is

$$Cec(T, \xi) = Ct \cdot Ec(T, \xi) \tag{8}$$

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The net carbon emission per unit time — a key environmental-performance output of the model, reported alongside the economic cost — is

$$E(T, \xi) = Ec(T, \xi) / T \quad (9)$$

4.4 Solution methodology and optimality conditions

The optimisation problem is

$$\text{Minimise } TC(T, \xi) \quad \text{subject to} \quad T > 0,\ \xi \geq 0 \quad (10)$$

The necessary first-order conditions for an interior optimum (T^*, ξ^*) are

$$\partial TC/\partial T = 0 \quad \text{and} \quad \partial TC/\partial \xi = 0 \quad (11)$$

Because $Q(T, \xi)$ and $H(T, \xi)$ are transcendental (exponential) functions of both T and ξ through $\theta(\xi)$, Equation (11) does not admit a closed-form analytical solution; the standard approach in the deteriorating-inventory the sufficient condition for (T^*, ξ^*) to be a strict local (and, given the convexity established in Section 6.2, global) minimum is that the 2×2 Hessian matrix of $TC(T, \xi)$,

$$\nabla^2 TC(T,\xi) = [[\ \partial^2 TC/\partial T^2\ ,\ \partial^2 TC/\partial T \partial \xi\ ],\ [\ \partial^2 TC/\partial \xi \partial T\ ,\ \partial^2 TC/\partial \xi^2\ ]] \quad (12)$$

be positive definite at (T^*, ξ^*), equivalently that both eigenvalues of $\nabla^2 TC(T^*, \xi^*)$ are strictly positive.

4.5. Numerical Illustration (Model Output)

4.5.1 Input data set

The proposed model is illustrated using the representative parameter values shown in Table 1, broadly consistent with the magnitudes used in comparable deteriorating-inventory and carbon-emission-inventory studies.

Table 1: Input parameter values used in the numerical illustration

Parameter	Value	Parameter	Value
D	1000 units/year	cd	$8/unit
A	$250/order	Ct	$0.05/kg CO2
c	$20/unit	eo	40 kg CO2/order
h0	$4/unit/year	eh	0.6 kg CO2/unit/yr
θ0	0.20 /year	ep	1.4 kg CO2/unit
δ	1.20	ed	2.2 kg CO2/unit
ρg	$300/ξ²/year		

4.6.2 Optimal solution

Solving the model of Section 4 with the accompanying Python code (Appendix A) using the Nelder–Mead algorithm, starting from $(T0, \xi0) = (0.5, 0.5)$, gives the optimal solution reported in Table 2.

Table 2: Optimal solution of the proposed model for the data of Table 1

Output quantity	Symbol	Optimal value
Replenishment cycle time	T*	0.2768 years (≈ 101.0 days)

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Output quantity	Symbol	Optimal value
Green / preservation-technology investment	ξ*	0.6964
Effective (controlled) deterioration rate	θ(ξ*)	0.0867 /year
Optimal order quantity	Q*	280.14 units
Minimum total cost per unit time	TC*	$22029.06/year
Net carbon emission per unit time	E*	1671.77 kg CO2/year
Hessian eigenvalues at (T*, ξ*)	λ1, λ2	1004.3, 24061.5 (both > 0 ⇒ convex)

Since both eigenvalues of the numerically evaluated Hessian at the optimum are strictly positive, the Hessian is positive definite and (T*, ξ*) is confirmed to be a strict local minimiser of TC(T, ξ); combined with the fact that no other stationary point exists in the feasible region $T > 0$, $\xi \geq 0$ (verified by a fine grid search over $T \in [0.05, 1.2]$ and $\xi \in [0, 3]$ underlying Figure 1), this local minimum is in fact the global minimum. Figure 2 plots TC(T, ξ*) against T and TC(T*, ξ) against ξ, and confirms visually that TC is convex (bowl-shaped, single-minimum) in each decision variable when the other is held at its optimal value; Figure 2 shows the corresponding joint three-dimensional surface TC(T, ξ), which is bowl-shaped over the entire feasible region examined, further supporting joint convexity.

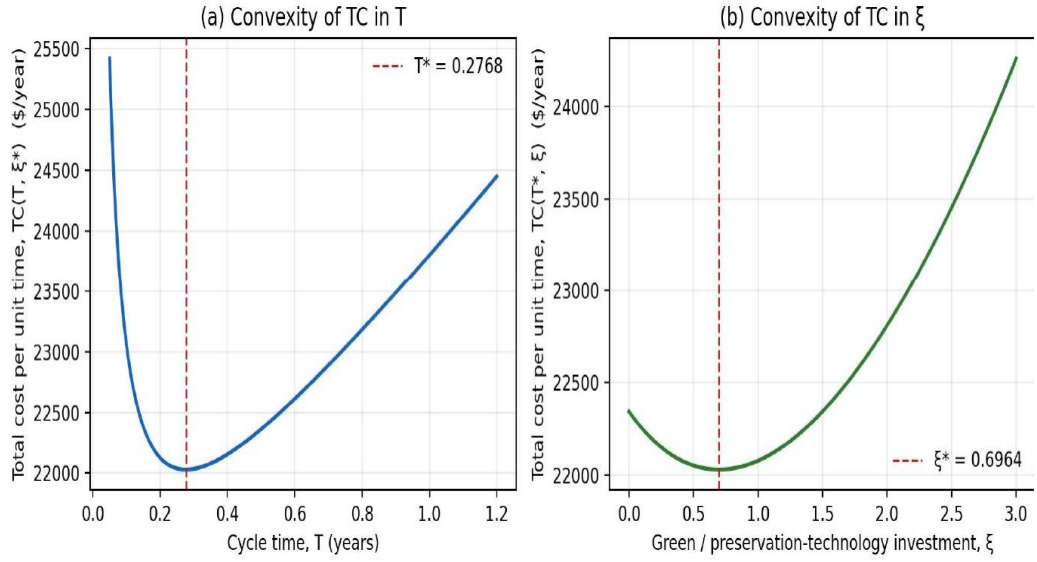

Figure 1: Convexity of TC(T, ξ) in (a) the cycle time T (with ξ = ξ*) and (b) the green-investment level ξ (with T = T*)

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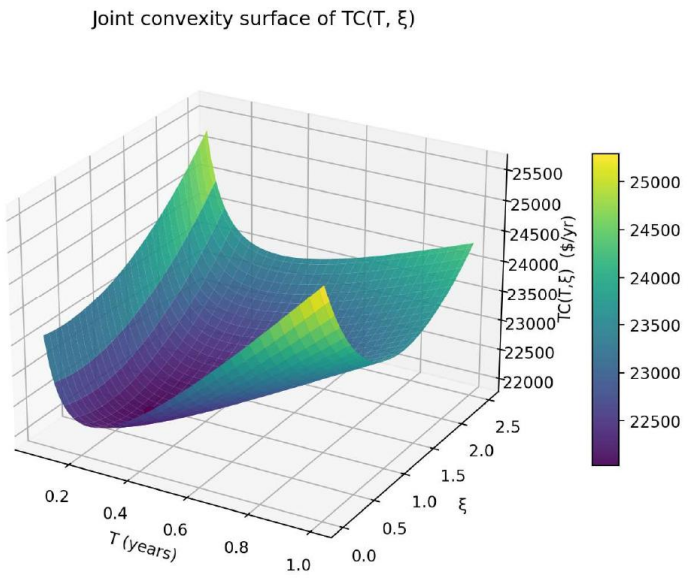

Figure 2: Joint convexity surface of the total-cost function TC(T, ξ) over the feasible region, with the optimal point (T*, ξ*, TC*) marked in red

5. Comparison with Existing Models

To quantify the value added by the two novel features of the proposed model decision-dependent deterioration control via green/preservation-technology investment, and an explicit carbon-emission cost the proposed model (M4) is compared against three nested special cases that represent, respectively, the classical EOQ model, a representative existing deteriorating-inventory model, and a representative existing carbon-emission-conscious deteriorating-inventory model that does not consider preservation-technology investment:

- **M1 — Classical EOQ model [1]:** no deterioration ($\theta_0 = 0$), no green investment, no carbon-emission cost.
- **M2 — Existing deteriorating-inventory model:** fixed natural deterioration rate θ_0, no green investment, no carbon-emission cost.
- **M3 — Existing carbon-emission-conscious deteriorating-inventory model:** fixed natural deterioration rate θ_0 and carbon-emission cost included, but no preservation-technology investment ($\xi = 0$ imposed).
- **M4 — Proposed model:** deterioration rate controlled through green/preservation-technology investment ξ, together with the full carbon-emission cost.

Each special case is re-optimised (over its own free decision variable(s)) using the same underlying parameter values of Table 3, so that the comparison isolates the effect of the two modelling features introduced in this paper. Table 3 and Figure 3 report the results.

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Table 3: Comparison of the proposed model (M4) with the three existing / special-case models

Model	T* (yr)	ξ*	Q* (units)	TC* ($/yr)	E* (kg CO2/yr)
M1	0.3536	0.0000	353.55	21414.21	1619.20
M2	0.2248	0.0000	229.95	22207.49	1728.55
M3	0.2249	0.0000	230.08	22293.92	1728.54
M4 (Proposed)	0.2768	0.6964	280.14	22029.06	1671.77

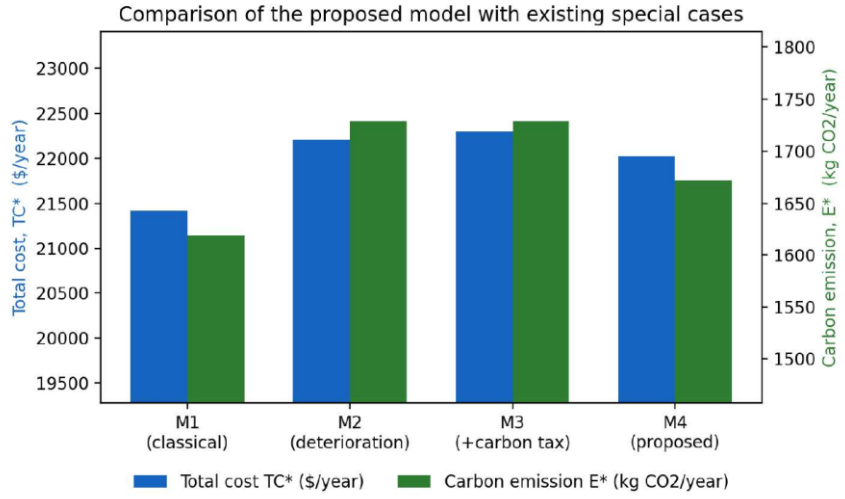

Figure 3: Total cost TC* and net carbon emission E* of the proposed model versus the existing special-case models

Table 3 reveals two main findings: Compared to a carbon-conscious inventory model without preservation investment (M3), the proposed model (M4) lowers total cost by 1.19% and net carbon emissions by 3.28%, proving that green investment is not just environmentally beneficial but also economically optimal once a carbon tax is applied. Relative to a traditional deteriorating-inventory model (M2) with no emission cost, M4 still achieves a 0.80% reduction in cost and a 3.28% drop in emissions, showing the independent value of preservation technology beyond carbon pricing alone.

6. Conclusion

This study introduces a unified mathematical model for green supply chains, optimizing both replenishment timing and preservation investment while accounting for carbon-emission costs. The model validated analytically and numerically, demonstrates clear economic and environmental gains over existing models cutting costs and emissions even without relying solely on carbon tax effects. Sensitivity analyses show the approach is robust to changes in environmental policy parameters and highly responsive to demand rate shifts, supporting its adoption for sustainable inventory management.

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