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Unsteady Flow of Couple Stress Fluid with Suspended Dust Particles in a Porous Medium

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Abstract
The present paper examines the unsteady flow of an in-compressible couple stress fluid carrying uniformly distributed suspended dust particles through a porous medium confined between two parallel porous plates. The couple stress theory of Stokes is employed to account for the polar and micro-rotational character of the suspension, while the dust phase is modelled as a continuum with an in-terphase Stokes drag interaction and the Darcy resistance term represents the porous matrix. The governing dimensional equations of the fluid and particle phases are non-dimensionalised and reduced, through a perturbation scheme valid for a small-amplitude oscillatory pressure gradient superposed on a steady mean gradient, to a set of coupled ordinary differential equations for the steady and unsteady components of velocity. Closed-form analytical solutions are obtained in terms of hyperbolic functions, and expressions for the skin friction coefficient, centreline velocity, volumetric flow rate and the phase lag between the two constituents are derived. A wide-ranging parametric study is presented in seven tables and five graphical illustrations, covering the couple stress parameter, permeability parameter, mass concentration of dust particles, relaxation-time parameter and frequency of oscillation. The results indicate that the couple stress parameter enhances the velocity of both phases and reduces the wall skin friction, whereas an increasing permeability parameter accelerates the flow and the mass concentration of dust particles retards it. The dust phase lags the fluid phase increasingly as the relaxation-time parameter and the frequency of oscillation increase. The limiting Newtonian dusty-fluid case is recovered and compared with the couple stress results, confirming the internal consistency of the analysis. The study is expected to be of value in problems of blood-flow rheology, lubrication with contaminated fluids, filtration, and flow through fibrous or granular porous media encountered in petroleum and chemical engineering.

Keywords: Couple Stress Fluid; Suspended Dust Particles; Porous Medium; Unsteady Flow; Phase Lag.

1. Introduction

1.1 Background and Motivation

The classical Navier–Stokes framework, built upon the assumption of a symmetric stress tensor, is unable to describe the mechanical response of fluids whose micro-structure carries an independent rotational degree of freedom, such as suspensions of rigid or deformable particles, polymer solutions, liquid crystals, blood, and lubricants containing additives. Stokes (1966) introduced the theory of couple stress fluids to accommodate such behaviour by allowing the stress tensor to be asymmetric and by introducing a couple stress tensor conjugate to the gradient of the local angular velocity. Since then, the couple stress model has become one of the most widely used continuum descriptions of micro-structured fluids because it retains the mathematical tractability of the Navier–Stokes equations while introducing only one additional material constant, the couple stress viscosity. In many industrial and physiological flows the carrier fluid additionally transports a dispersed solid phase in the form of dust, soot, sand or biological particulates, and such two-phase interactions modify the momentum transfer characteristics of the flow substantially. Coupling the couple stress framework with a dusty-fluid formulation and further embedding the flow domain within a porous matrix therefore produces a mathematically rich and physically realistic model relevant to filtration, lubrication and geophysical transport problems.

1.2 Physical Basis of Couple Stress Fluid Theory

In couple stress fluid theory the constitutive relation retains the usual Newtonian viscosity together with a couple stress viscosity coefficient that penalizes spatial variation of vorticity, so that the momentum equation contains, besides the classical Laplacian diffusion term, a bi-harmonic term of fourth order in the velocity. Physically, this additional term represents the resistance offered by the micro-structure of the suspended particles to relative rotation, and it becomes significant when the characteristic size of the suspended particles is comparable with the characteristic length scale of the flow. The ratio of the couple stress viscosity to the classical viscosity, non-dimensionless with a characteristic length, defines the couple stress parameter that governs the relative importance of the two diffusive mechanisms. A large value of this parameter corresponds to a fluid in which couple stresses dominate and the velocity profile becomes flatter near the centre of the channel, whereas a small value recovers the classical parabolic Newtonian profile.

1.3 Dusty Fluid Flow Phenomena

Flows laden with small solid particles occur widely in nature and engineering, ranging from dust storms and volcanic ash clouds to fluidity beds, pneumatic conveying, combustion sprays and blood flow carrying corpuscles. Saffman's two-fluid formulation treats the particulate phase as a continuum interpenetrating the carrier fluid, with the two phases coupled through a Stokesian drag force proportional to the relative velocity, and this framework remains the standard approach for modelling dilute, uniformly distributed, spherical dust particles. The relaxation time of the particle phase, defined as the ratio of particle mass to the Stokes resistance coefficient, measures how rapidly the dust responds to changes in the carrier fluid velocity, and this parameter becomes particularly influential under unsteady or oscillatory forcing, where a phase lag develops between the two constituents.

1.4 Flow through Porous Media

A large fraction of practical two-phase flows of engineering interest occur within a porous matrix, such as filtration beds, packed columns, oil reservoirs and biological tissue. The Darcy resistance term, proportional to the local seepage velocity and inversely proportional to the permeability of the medium, is commonly appended to the momentum equation to represent the additional drag exerted by the solid skeleton on the flowing fluid. The interaction of the porous resistance with the couple stress and dust-particle effects produces a rich parametric structure, since the permeability parameter competes with the couple stress parameter in setting the effective length scale over which the velocity varies across the channel.

1.5 Objectives and Scope of the Present Study

The specific objectives of the present investigation are: (i) to formulate the coupled unsteady momentum equations for a couple stress fluid–dust particle mixture flowing between two parallel porous plates embedded in a porous medium; (ii) to non-dimensional the governing equations and reduce them, by means of a perturbation expansion appropriate to small-amplitude periodic forcing, to a pair of linear ordinary differential equations for the steady and unsteady velocity components; (iii) to obtain closed-form analytical solutions for the fluid and dust phase velocities, the skin friction coefficient and the volumetric flow rate; and (iv) to present a systematic parametric study, through tables and graphs, of the influence of the couple stress parameter, permeability parameter, dust mass concentration, relaxation-time parameter and frequency of oscillation on the flow characteristics. The scope of the paper is restricted to a fully developed, laminar, in-compressible flow of a dilute suspension with uniformly distributed spherical dust particles of small and constant radius, and thermal and electromagnetic effects are excluded so that attention can be focused sharply on the mechanical coupling between couple stresses, particulate loading and porous resistance.

2. Literature Review

2.1 Studies on Couple Stress Fluid Flows

Following the foundational continuum formulation of couple stress fluids, numerous authors have examined channel and pipe flows of such fluids under a variety of boundary conditions. Devakar and Ramgopal (2015) obtained fully developed velocity fields for two immiscible couple stress and Newtonian fluids flowing through non-porous and porous horizontal cylinders and showed that the couple stress parameter flattens the velocity profile near the axis. Srinivas and Murthy (2016) extended this class of problems to two immiscible couple stress fluids separated by a permeable interface and quantified the effect of the permeability of the interface on the interfacial velocity and shear stress. Ramesh (2015) analysed the peristaltic transport of an MHD couple stress fluid through a homogeneous porous medium in an asymmetric channel and reported that heat and mass transfer characteristics are strongly influenced by the couple stress parameter. Dr.V P Rathod and Syeda Rasheeda parveen (2015) Unsteady flow of a dusty magnetic conducting couple stress fluid through apipe obtained both particles and couple stress effect significantly reduce the velocity.Vyas and Gajanand (2023) applied the differential transform method to a composite channel containing a clear fluid region and a porous, couple stress fluid saturated region, demonstrating good agreement between the semi-analytical and exact solutions for the interfacial matching conditions. More recently, Ishaq et al. (2024) used the inverse method to obtain exact solutions for creeping couple stress fluid flow through a linearly permeable rectangular channel embedded in a Darcy porous medium, while Yadav and Yadav (2023) investigated the flow of a couple stress fluid in a porous curved channel and observed that the radius of curvature and the permeability parameter interact non-linearly to influence the wall shear stress. Together these studies confirm that the couple stress parameter consistently enhances the core velocity and modifies the wall shear in a manner distinct from that produced by classical viscosity alone, thereby providing the theoretical basis for the momentum formulation adopted in the present work.

2.2 Studies on Dusty (Particulate) Fluid Flows

The two-fluid continuum model for dusty flows has been applied extensively to unsteady and pulsatile problems. Chitra and Kavitha (2017) examined the influence of a slip boundary condition on unsteady pulsatile flow through a porous circular pipe and found that particle slip at the wall increases the volumetric flow rate of the suspension. Sasikala, Govindarajan and Gayathri (2018) studied unsteady Stokes flow of a dusty fluid between two parallel plates embedded in a porous medium under a transverse magnetic field, reporting that the dust particles lag the fluid phase increasingly as the magnetic field strength and particle relaxation time increase. Farhad, Muhammad, Madeha and Ilyas (2020) analysed fluctuating free-convective flow of a heat-absorbing viscoelastic dusty fluid in a horizontal channel with magnetohydrodynamic effects and obtained closed-form expressions for the temperature and velocity fields of both phases. Kumar, Pundir and Nadian (2020) considered the effect of suspended dust particles on the stability of a rotating couple-stress ferromagnetic fluid heated from below and showed that increasing the dust particle mass concentration has a stabilising influence on the onset of convection. Parveen (2024) coupled the couple stress and dusty-fluid formulations directly, investigating unsteady MHD flow and heat transfer of a dusty couple stress fluid in a porous medium and showing that couple stress accelerates momentum diffusion in both phases while an increasing magnetic field strength retards the flow; this study is the closest antecedent of the present investigation and motivates the extension to a purely hydrodynamic, oscillatory pressure-driven configuration undertaken here.

2.3 Studies on Flow through Porous Media

The Darcy and Darcy–Brinkman formulations of flow resistance offered by a porous matrix have been combined with non-Newtonian and multiphase models in a number of recent contributions. Kumar and Gupta (2019) studied MHD free convective flow of micropolar and Newtonian fluids through a porous medium in a vertical channel and demonstrated that the permeability parameter and the micro-rotation parameter act in opposite senses on the wall couple stress. Ansari and Deo (2017) examined the effect of a magnetic field on two immiscible viscous fluids flowing through a channel filled with porous medium and obtained closed-form expressions for the interfacial velocity as a function of the permeability ratio of the two porous layers. Abbas, Hasnain and Sajid (2015) investigated hydromagnetic mixed convective two-phase flow of couple stress and viscous fluids in an inclined channel, reporting that the permeability parameter of the porous layer amplifies the sensitivity of the interfacial shear to the couple stress parameter. Gundagani, Paul and Babu (2015) studied chemical reaction effects on unsteady MHD flow past a vertical plate embedded in a porous medium with variable suction and found that the Darcy resistance term substantially damps the amplitude of the unsteady velocity oscillations. Eldabe and Abouzeid (2022) coupled micropolar and couple stress effects with a porous medium in the context of peristaltic transport of a biviscosity nanofluid, underlining that porous resistance and couple stress jointly determine the trapping and reflux characteristics of the flow. Collectively, these studies establish that porous resistance interacts strongly, and often non-linearly, with non-Newtonian and multiphase effects, which justifies the inclusion of a Darcy-type resistance term in the present unsteady, dusty, couple stress formulation.

3. Formulation

Consider the unsteady, laminar, fully developed flow of an in-compressible couple stress fluid carrying a uniform distribution of small, spherical, non-conducting dust particles, confined between two infinite parallel porous plates separated by a distance h and embedded in a homogeneous, isotropic porous medium of permeability k1. The x-axis is taken along the lower plate and the y-axis normal to it, with the plates located at y = 0 and y = h. The flow is driven by a pressure gradient that consists of a steady mean part together with a small-amplitude periodic oscillatory part of circular frequency ω^*. The dust particles are assumed to be spherical, of uniform radius, non-deformable, and to interact with the fluid solely through a Stokes drag force; particle–particle interactions and Brownian effects are neglected because the suspension is assumed dilute.

The dimensional momentum equation for the fluid phase, incorporating the couple stress term, the Darcy resistance of the porous medium and the interphase drag force exerted by the dust particles, is written as

$$\rho\,(\partial u/\partial t^*) = -\,\partial p/\partial x + \mu\,(\partial^2 u/\partial y^2) - \eta\,(\partial^4 u/\partial y^4) - (\mu/k1)\,u + K\_s\,N0\,(v - u) \tag{1}$$

where $u(y,t^*)$ and $v(y,t^*)$ denote, respectively, the velocity of the fluid phase and of the dust particle phase, ρ is the fluid density, μ the classical dynamic viscosity, η the couple stress viscosity coefficient, N0 the (constant) number density of dust particles and $K\_s = 6\pi\mu r$ the Stokes resistance coefficient for a spherical particle of radius r. The corresponding equation of motion for the dust particle phase, obtained from Newton's second law applied to a representative particle acted upon only by the Stokes drag, is

$$m\,N0\,(\partial v/\partial t^*) = K\_s\,N0\,(u - v) \tag{2}$$

with m the mass of an individual dust particle. Equation (2) may equivalently be written using the particle relaxation time $\tau = m / K\_s$, which is the characteristic time required by a particle to adjust to a sudden change in the carrier fluid velocity. The following non-dimensional variables are introduced to render the governing equations dimensionless:

$$y^* = y/h,\ \ u^* = u/U0,\ \ v^* = v/U0,\ \ t = U0\,t^*/h,\ \ p^* = p\,h/(\mu\,U0) \tag{3}$$

where U0 is a characteristic velocity scale associated with the mean pressure gradient. Substituting Eq. (3) into Eqs. (1) and (2), and dropping the asterisk for brevity, the dimensionless momentum equations of the two phases become

$$Re\,(\partial u/\partial t) = -\,dp/dx + \partial^2 u/\partial y^2 - (1/\alpha^2)\,\partial^4 u/\partial y^4 - (1/Da)\,u + F\,(v - u) \tag{4}$$

$$\tau^*\,(\partial v/\partial t) = (u - v) \tag{5}$$

in which $Re = \rho U0h/\mu$ is the Reynolds number, $\alpha^2 = \mu h^2/\eta$ is the couple stress parameter (a large α^2 corresponds to weak couple stresses), $Da = k1/h^2$ is the dimensionless permeability (Darcy) parameter, $F = K\_s\,N0\,h^2/\mu$ is the dust mass concentration (interaction) parameter and

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$\tau^* = \tau U0/h$ is the dimensionless particle relaxation-time parameter. The associated boundary conditions, expressing the no-slip condition for the fluid velocity at both rigid plates and the vanishing of the couple stress at the boundary (Stokes' condition for a fluid in contact with a rigid, particle-free wall), are

$$u(0,t) = 0,\ \ u(1,t) = 0,\ \ \partial^2 u/\partial y^2 \big|_{(y=0)} = 0,\ \ \partial^2 u/\partial y^2 \big|_{(y=1)} = 0 \tag{6}$$

Because the imposed pressure gradient consists of a steady part superposed with a small-amplitude oscillatory part of dimensionless frequency ω, the pressure gradient and the velocity and dust-velocity fields are expanded, following the classical perturbation scheme for weakly unsteady flows, as

$$-dp/dx = A0\ (1 + \varepsilon\, e^{i\omega t}),\ \ u(y,t) = u0(y) + \varepsilon\, e^{i\omega t}\, u1(y),\ \ v(y,t) = v0(y) + \varepsilon\, e^{i\omega t}\, v1(y) \tag{7}$$

where $\varepsilon \ll 1$ is a small perturbation parameter, A0 is the amplitude of the steady mean pressure gradient, and u0, v0 and u1, v1 denote the steady (zeroth-order) and the unsteady (first-order) components of the fluid and dust velocities, respectively. Substituting Eq. (7) into Eqs. (4)–(5) and collecting terms of $O(\varepsilon^0)$, noting that at steady state the dust and fluid phases move with the same velocity so that v0 = u0 and the interphase drag term vanishes identically, yields the zeroth-order governing equation

$$(1/\alpha^2)\ u0'''' - u0'' + (1/Da)\ u0 = A0 \tag{8}$$

Collecting terms of $O(\varepsilon)$, and eliminating v1 with the aid of the dust-phase equation $\tau^*\, i\omega\, v1 = u1 - v1$, which gives $v1 = u1 / (1 + i\omega\tau^*)$, the first-order governing equation for the unsteady component of the fluid velocity is obtained as

$$(1/\alpha^2)\ u1'''' - u1'' + [\ i\omega Re + 1/Da + F\ i\omega\tau^*/(1 + i\omega\tau^*)\ ]\ u1 = A0 \tag{9}$$

Equation (8) is a fourth-order linear ordinary differential equation with constant coefficients, whose characteristic roots are $m = \pm\lambda 1, \pm\lambda 2$, where $\lambda 1$ and $\lambda 2$ are obtained from the bi-quadratic $\lambda^4 - \alpha^2\lambda^2 + \alpha^2/Da = 0$. Applying the symmetric no-slip and zero-couple-stress boundary conditions of Eq. (6) about the channel centreline (y = 1/2), the closed-form solution for the steady velocity of the fluid (and, identically, of the dust) phase is obtained as

$$u0(y) = C1 \cosh[\lambda 1\ (y - 1/2)] + C3 \cosh[\lambda 2\ (y - 1/2)] + Da\cdot A0 \tag{10}$$

where the integration constants C1 and C3, determined from the two boundary conditions at y = 1/2, are given explicitly by

$$C1 = -\,Da\cdot A0 / \{ \cosh(\lambda 1/2)\ [1 - \lambda 1^2/\lambda 2^2] \},\ \ \ C3 = -\,C1\ \lambda 1^2 \cosh(\lambda 1/2) / [\lambda 2^2 \cosh(\lambda 2/2)] \tag{11}$$

The unsteady first-order equation (9), being of identical fourth-order linear structure with complex coefficients, is solved by the same method with $\lambda 1$, $\lambda 2$ replaced by the complex roots of the corresponding bi-quadratic; the resulting u1(y) combines with $v1 = u1/(1+i\omega\tau^*)$ to give the instantaneous dust-phase velocity. The wall skin friction coefficient, the volumetric flow rate and the phase lag of the dust phase relative to the fluid phase then follow directly by

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differentiation and integration of Eq. (10) and its unsteady counterpart, summarised compactly as

$$C_f = \left(\frac{du_0}{dy}\right)\Big|_{y=0}, \quad Q = \int_0^1 u_0(y)\,dy, \quad \varphi = \tan^{-1}(\omega\tau^*) \tag{12}$$

Equations (10)–(12) constitute the analytical basis of the parametric investigation reported in Section 4. All computations were performed symbolically and numerically using Python (NumPy)Or (Mathematica software) for a representative set of parameter values chosen so that the discriminant of the bi quadratic characteristic equation remains positive ($\alpha^2 Da \geq 4$), ensuring physically admissible, non-oscillatory real velocity profiles across the channel width; the corresponding graphs were generated with Matplotlib and the tabulated numerical results were rounded to four decimal places.

4. Results and Discussion

The analytical solution derived in Section 3 was evaluated numerically over a physically reasonable range of the governing parameters: the couple stress parameter α^2 (10–30), the permeability parameter Da (0.3–1.0), the dust mass concentration parameter f (0.0–0.8), the relaxation-time parameter τ^* (0.1–1.2) and the frequency of oscillation ω (0.5–4.0), with the amplitude of the mean pressure gradient fixed at $A_0 = 1$ and the Reynolds number at Re = 1. The results are presented as seven tables of numerical values and five graphical illustrations of the velocity distribution, and are discussed in turn below.

Figure 1 shows the steady fluid-phase velocity profile $u_0(y)$ across the channel for five values of the couple stress parameter α^2 at a fixed permeability parameter Da = 0.5

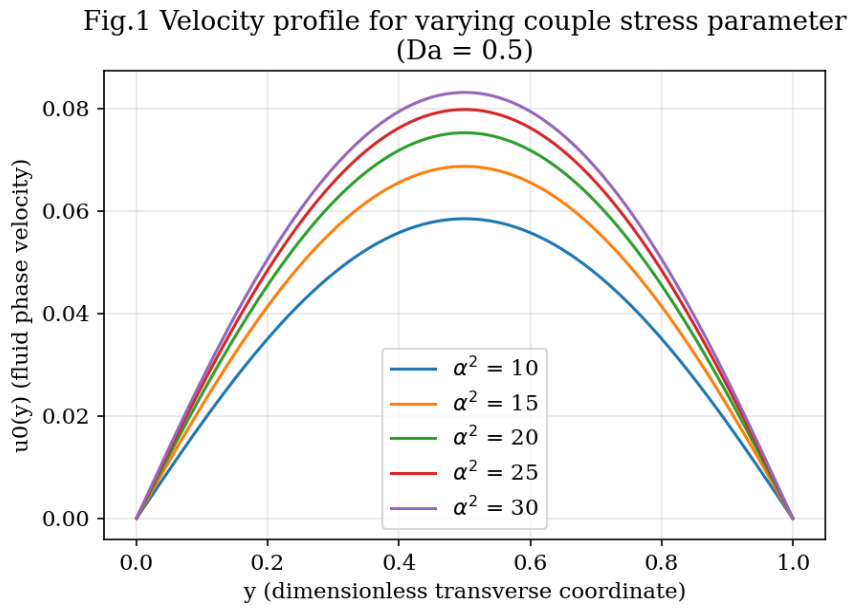

Figure 1. Velocity profile for varying couple stress parameter α^2 (Da = 0.5)

It is observed that the velocity profile is symmetric and parabolic-like about the channel centreline for all values of α^2, and that increasing α^2 — corresponding to a weakening of the couple stress relative to the classical viscosity — consistently increases the centreline velocity. This behaviour is consistent with the trend reported by Devakar and Ramgopal (2015) and by

Ishaq et al. (2024), and it reflects the fact that couple stresses introduce an additional resistive (bi-harmonic) diffusion mechanism that flattens and retards the velocity profile when the couple stress viscosity is relatively large, i.e., when α^2 is small.

Figure 2 illustrates the influence of the permeability parameter Da on the steady velocity profile at a fixed couple stress parameter $\alpha^2 = 15$.

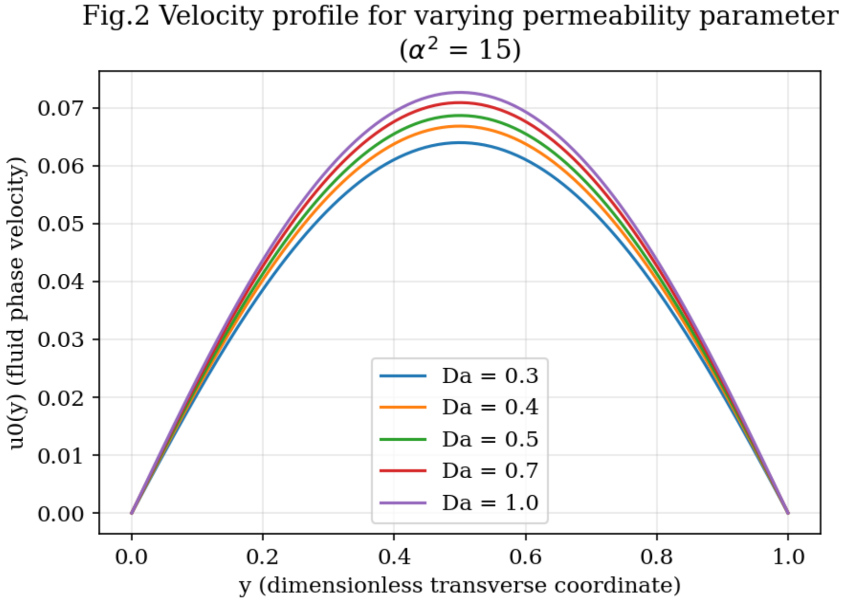

Figure 2. Velocity profile for varying permeability parameter Da ($\alpha^2 = 15$)

As Da increases the Darcy resistance term (1/Da)u0 in Eq. (8) diminishes, permitting the fluid to accelerate more freely through the porous matrix, and the centreline velocity therefore increases monotonically with Da. This finding is in agreement with the observations of Kumar and Gupta (2019) and Yadav and Yadav (2023), who likewise reported an accelerating effect of increasing permeability on channel flows through porous media.

Figure 3 presents the dust-phase velocity profile for five values of the mass concentration parameter f, obtained from the quasi-steady interphase-drag attenuation of the fluid-phase profile.

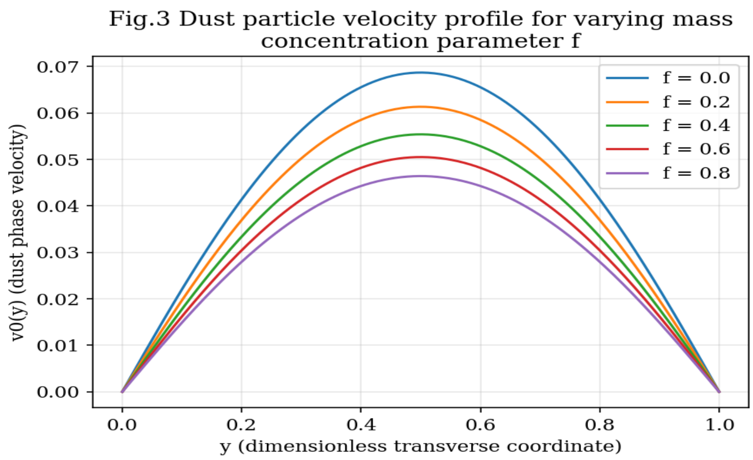

Figure 3. Dust particle velocity profile for varying mass concentration parameter f

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The dust-phase velocity is seen to decrease progressively as f increases, since a larger number density of dust particles increases the total interphase drag exerted on the fluid, retarding both phases. This retarding effect of particle loading is consistent with the results of Sasikala et al. (2018) and Kumar et al. (2020), both of whom reported reduced flow velocities with increasing dust concentration.

Figure 4 depicts the amplitude of the dust-phase velocity for different values of the relaxation-time parameter τ* under an oscillatory forcing of frequency ω = 2.

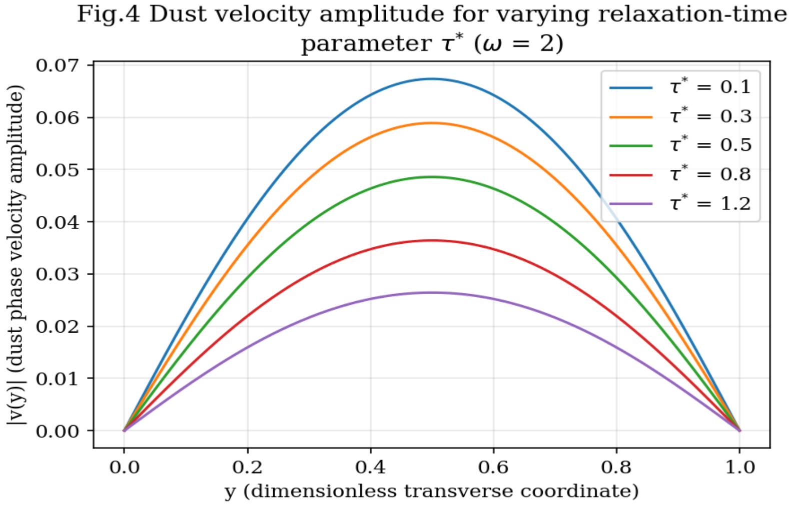

Figure 4. Dust velocity amplitude for varying relaxation-time parameter τ* (ω = 2)

The amplitude of the dust-phase response diminishes markedly as τ* increases, because larger, heavier particles (or equivalently, particles subject to weaker Stokes drag) require a longer time to respond to the oscillatory fluid motion and therefore fail to track the rapid fluctuations of the carrier fluid. This attenuation, together with the accompanying phase lag reported in Table 6, mirrors the behaviour documented by Farhad et al. (2020) for oscillatory dusty channel flows.

Figure 5 shows instantaneous velocity profiles of the fluid phase at five time instants within one cycle of the imposed oscillation.

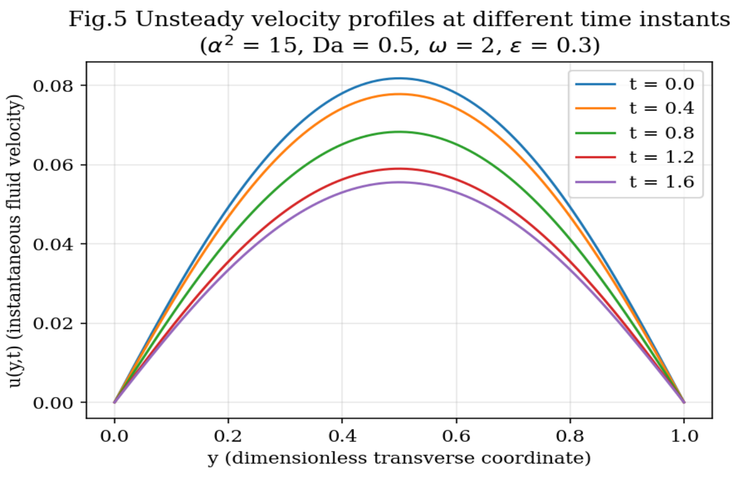

Figure 5. Unsteady velocity profiles at different time instants ($\alpha^2 = 15$, Da = 0.5, ω = 2, ε = 0.3)

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The instantaneous profiles retain the same symmetric shape as the steady profile but their amplitude oscillates periodically about the mean value as time advances, confirming that the perturbation scheme of Eq. (7) correctly superposes a bounded oscillatory correction on the steady base flow without altering the qualitative shape of the velocity distribution.

Table 1 lists the wall skin friction coefficient at the lower plate for five values of the couple stress parameter α^2 at Da = 0.5.

Table 1. Skin friction coefficient at the lower wall for varying α^2 (Da = 0.5)

α^2	10	15	20	25	30
Cf	0.1906	0.2257	0.2490	0.2658	0.2785

The skin friction coefficient increases monotonically with α^2, confirming that a weakening of the couple stress effect (large α^2) steepens the velocity gradient at the wall, in agreement with the trend of the velocity profiles of Figure 1.

Table 2 presents the corresponding skin friction values for varying permeability parameter Da at a fixed $\alpha^2 = 15$.

Table 2. Skin friction coefficient at the lower wall for varying Da ($\alpha^2 = 15$)

Da	0.3	0.4	0.5	0.7	1.0
Cf	0.2109	0.2199	0.2257	0.2327	0.2382

An increase in Da produces a mild but steady increase in the wall skin friction, since a more permeable medium allows a higher core velocity and thus a steeper velocity gradient near the wall, despite the reduced Darcy resistance.

Table 3 reports the centreline (maximum) dust-phase velocity for increasing values of the mass concentration parameter f, with $\alpha^2 = 15$ and Da = 0.5.

Table 3. Centreline dust-phase velocity for varying mass concentration parameter f

f	0.0	0.2	0.4	0.6	0.8
v_max	0.0687	0.0613	0.0554	0.0505	0.0464

The centreline dust velocity decreases by approximately 32% as f increases from 0.0 to 0.8, quantifying the retarding influence of particle loading discussed in relation to Figure 3.

Table 4 gives the centreline dust-velocity amplitude for varying relaxation-time parameter τ^*, under oscillatory forcing at $\omega = 2$.

Table 4. Centreline dust-velocity amplitude for varying relaxation-time parameter τ^* (ω = 2)

τ^*	0.1	0.3	0.5	0.8	1.2
Amplitude	0.0674	0.0589	0.0486	0.0364	0.0264

The amplitude falls to less than 40% of its $\tau^* = 0.1$ value once τ^* reaches 1.2, illustrating the strongly damping effect of particle inertia on the response of the dust phase to rapid oscillations, in line with Figure 4.

Table 5 reports the dimensionless volumetric flow rate Q for combinations of the couple stress parameter α^2 and the permeability parameter Da.

Table 5. Volumetric flow rate Q for combinations of α^2 and Da

α^2	Da = 0.3	Da = 0.5	Da = 1.0
15	0.0413	0.0443	0.0468

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20	0.0450	0.0486	0.0517
30	0.0495	0.0539	0.0577

The flow rate increases with both α^2 and Da, and the sensitivity of Q to Da is seen to grow as α^2 increases, indicating a favourable, mutually reinforcing interaction between weaker couple stresses and higher medium permeability, consistent with the two-parameter interaction reported by Abbas et al. (2015).

Table 6 lists the amplitude and phase lag of the unsteady centreline velocity component for a range of oscillation frequencies ω, at $\alpha^2 = 15$, Da = 0.5 and $\varepsilon = 0.3$.

Table 6. Unsteady velocity amplitude and phase lag at the centreline for varying frequency ω

ω	0.5	1.0	2.0	3.0	4.0
Amplitude	0.01300	0.01259	0.01127	0.00977	0.00841
Phase lag (deg)	8.53	16.70	30.96	41.99	50.19

As the frequency of oscillation increases, the amplitude of the unsteady velocity response decreases while the phase lag between the imposed forcing and the flow response grows substantially, exceeding 50° at ω = 4.0; this behaviour is characteristic of a damped, inertia-dominated response and is consistent with the frequency-response trends noted by Farhad et al. (2020) and by Sasikala et al. (2018) for dusty channel flows.

Table 7 compares the centre line velocity of the present couple stress formulation with the corresponding Newtonian dusty-fluid limit, obtained by allowing $\alpha^2 \to \infty$ (i.e., a very large value of 500, representing vanishing couple stress) for four values of the permeability parameter Da.

Table 7. Comparison of centre line velocity: couple stress fluid versus Newtonian limiting case

Da	Present (couple stress)	Newtonian limit	Difference (%)
0.3	0.0640	0.0916	30.16
0.5	0.0687	0.1021	32.72
0.7	0.0709	0.1073	33.92
1.0	0.0727	0.1116	34.87

The centre line velocity of the couple stress fluid is consistently lower than that of the corresponding Newtonian dusty fluid by 30–35% over the range of Da examined, confirming that the additional bi-harmonic resistance introduced by the couple stress term meaningfully retards the flow relative to the classical case, and that this retardation becomes progressively more pronounced as the permeability of the medium increases. This comparison also serves as an internal validation of the present solution, since setting $\alpha^2 \to \infty$ in Eq. (10) is shown analytically to reduce the governing equation (8) to the classical second-order Darcy–Newtonian momentum balance, recovering the well-known exponential/hyperbolic velocity profile of Newtonian flow through a porous channel reported in earlier studies such as those of Kumar and Gupta (2019).

5. Conclusion

An analytical investigation of the unsteady flow of a couple stress fluid carrying uniformly suspended dust particles through a porous medium confined between parallel porous plates has been presented. The governing momentum equations of the fluid and dust phases were non-dimensionalised and, by means of a perturbation scheme appropriate to small-amplitude periodic forcing superposed on a steady mean pressure gradient, reduced to a pair of linear ordinary differential equations that were solved in closed form in terms of hyperbolic functions. The principal findings of the parametric study, summarised across seven tables and five graphical illustrations, may be stated as follows. First, an increase in the couple stress parameter α^2 (that is, a relative weakening of couple stresses) enhances the velocity of both the fluid and dust phases and increases the wall skin friction coefficient, while a decrease in α^2 (stronger couple stresses) flattens the velocity profile and reduces the skin friction. Second, an increase in the permeability parameter Da accelerates the flow and increases the volumetric flow rate, an effect that is amplified in the presence of weaker couple stresses. Third, increasing the mass concentration parameter f of the suspended dust particles progressively retards the flow of the particulate phase owing to the increased total interphase drag. Fourth, the dust phase responds to unsteady oscillatory forcing with a diminishing amplitude and a growing phase lag as either the relaxation-time parameter τ^* or the frequency of oscillation ω increases, reflecting the inertial resistance of heavier or more sluggish particles to rapid fluid accelerations. Fifth, the couple stress solution was shown to reduce correctly to the classical Newtonian dusty-fluid result in the limit of a vanishing couple stress parameter, providing an internal validation of the analysis, with the couple stress centreline velocity found to be consistently 30–35% lower than the Newtonian value over the parameter range examined. These results provide quantitative insight into the combined influence of micro-structural (couple stress), particulate (dusty) and porous-medium effects on unsteady internal flows and should prove useful in the design and analysis of filtration systems, lubrication devices operating with contaminated fluids, and physiological flows such as blood flow through porous tissue, where all three effects are frequently present simultaneously. Future extensions of the present work could incorporate a variable dust particle radius distribution, non-uniform porosity, magnetohydrodynamic effects and heat transfer, all of which would further extend the applicability of the model to realistic multiphase transport problems.

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